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Theory of Spherical Agglomeration. I. Continuity Equation for Granules in a Rotating Conical Drum*

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Abstract

A theoretical study is made of the steady-state spherical agglomeration process in a rotating conical drum used to separate bitumen from solid particles in oil sands. Some consequences of the continuity equation for the flow of granules suspended in a bitumen-solvent liquid are investigated. From this equation, and experimental results, some general conclusions are reached concerning the distribution of granule sizes, their segregation, and their rates of production and destruction in the agglomeration process in the conical drum. It is argued that layering is a special form of coalescence.

INTRODUCTION

Some theoretical aspects of spherical agglomeration are considered here with particular reference to the recovery of bitumen from oil sands. In this separation method, which is described in a series of papers (*1–3*), finely divided sand particles suspended in a bitumen-solvent liquid are agglomerated into granules and separated from the liquid by suitable agitation and the addition of a small amount of water, which preferentially wets the sand and is immiscible with the suspending medium. The agglomeration process is an example of two-phase flow where one phase is a continuous fluid and the other phase consists of a high

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concentration of dispersed particles. Such flow has been studied theoretically by many authors (*4–9*) but in the absence of any particle aggregation. In this paper we consider the continuity equation for the granules (particles) undergoing coalescence, layering, and crushing in the agglomeration process. In particular, we confine our attention to steady-state flow in the rotating conical drum which has been used to separate bitumen from solid particles in oil sands (*1–3*). The general properties of the distribution of granules, their size segregation, and their rates of production and destruction in the conical drum will be investigated.

CONTINUITY EQUATION FOR MOTION OF GRANULES

We can consider a discrete or a continuous distribution of granule sizes. Choosing the latter, let $n(m)dm$ be the number of granules per unit volume in the mass range $m,m + dm$ and $\mathbf{v}(m)$ the velocity of the center of a granule of mass m. The continuity equation for granules of mass m is

$$\frac{\partial n(m)}{\partial t} + \nabla \cdot (n(m)\mathbf{v}(m)) = S(m) \quad (1)$$

where $S(m)$ is the source term due to the creation and destruction of granules of mass m in the agglomeration process. Precisely, $S(m)dm$ is the rate of change with time due to agglomeration in the number of granules per unit volume in the mass range $m,m + dm$. The quantities $n(m)$, $\mathbf{v}(m)$, and $S(m)$ all depend on position in the agglomerate charge contained in the rotating conical drum and also on time in the unsteady state. We wish to relate Eq. (1) to the macroscopic continuity condition for the dispersed phase in the continuum theory of two-phase flow. Since $mn(m)dm$ is the mass of the granules in unit volume in the mass range $m,m + dm$, the local total mass of granules in unit volume is

$$(1 - \varepsilon)\rho_d = \int mn(m)dm \quad (2)$$

where ρ_d is the density of a granule (which need not be constant) and ε is the mean fraction of the local volume occupied by the continuous (bitumen-solvent medium) phase, i.e., ε is the porosity. The local mass average velocity of the granules $\mathbf{v}_d$ is defined by

$$(1 - \varepsilon)\rho_d\mathbf{v}_d = \int mn(m)\mathbf{v}(m)dm \quad (3)$$

The quantities $n(m)$, $\mathbf{v}(m)$, $S(m)$, ε, and $\mathbf{v}_d$ are examples of averages over volumes large compared with a single granule volume but small on the scale of the physical apparatus. Such averages are characteristic of the parameters in the equations describing two-phase flow.

On multiplying by m and integrating Eq. (1) over all values of m, we obtain

$$\int m \frac{\partial n(m)}{\partial t} dm + \int m \nabla \cdot (n(m)\mathbf{v}(m))dm = \int mS(m)dm \quad (4)$$

We can interchange the orders of integration with respect to mass m and of differentiations with respect to time and space to obtain

$$\frac{\partial}{\partial t}[(1 - \varepsilon)\rho_d] + \nabla \cdot [(1 - \varepsilon)\rho_d \mathbf{v}_d] = \int mS(m)dm \quad (5)$$

making use of Eqs. (2) and (3). In the steady state, we have two relations:

$$\int mS(m)dm = 0 \quad (6)$$

and

$$\nabla \cdot [(1 - \varepsilon)\rho_d \mathbf{v}_d] = 0 \quad (7)$$

so that the three terms in Eq. (5) are separately zero. Equation (6) alone, which states that the rate of increase due to agglomeration of the total mass of granules in unit volume is zero, is necessary but not sufficient to ensure steady-state conditions. This is because the degree of trapping of the continuous (bitumen-solvent medium) phase by the granules will change during agglomeration so that both the granule density ρ_d and porosity ε may be time-dependent.

To illustrate the meaning of Eq. (6), consider a thin slab between parallel planes at z and $z + \Delta z$, normal to the axis of the rotating conical drum, where z is distance along this axis (Fig. 1). In the agglomeration process, small granules enter the slab through the plane z and agglomerate to form larger granules which leave the slab through the plane $z + \Delta z$. Fewer small granules but more larger granules leave at $z + \Delta z$, keeping the total mass of granules constant in the slab. It follows that a stable steady-state condition also leads to size segregation. Consider now a *horizontal* rotating, cylindrical drum in which agglomeration is taking

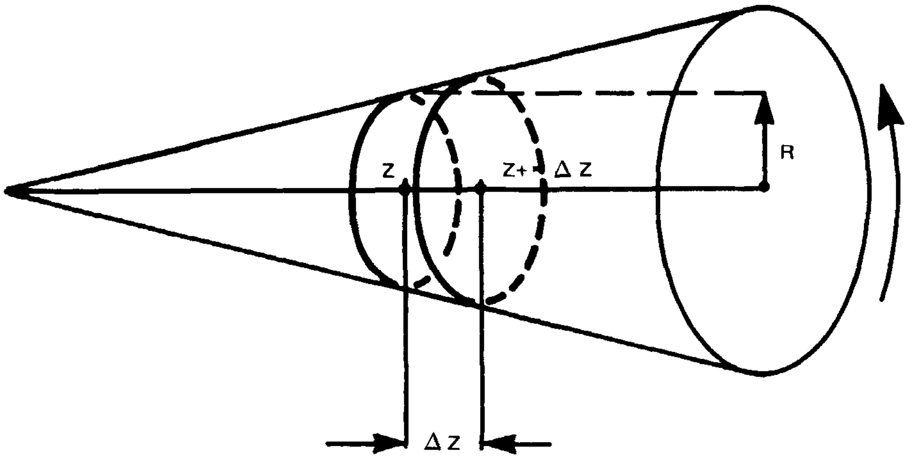

FIG. 1. Cylindrical polar coordinates R, θ, z. Thin slab of thickness Δz normal to cone axis.

place and any end effects can be neglected. In a steady-state condition the distribution of granules in this drum should be homogeneous which is impossible if agglomeration is taking place, since size segregation would result, producing unsteady conditions. The axis of the cylindrical drum must be tilted to the horizontal to make steady-state conditions possible. It is well known that continuous flow agglomeration in a horizontal or nearly horizontal rotating cylindrical drum is unstable, being characterized by "surging" or "cycling" (*10, 11*).

An example of the condition Eq. (6) is provided by coalescence of granules as follows. We assume that the source term is

$$S(m) = \tfrac{1}{2}\int_0^m \beta(m',m-m')n(m')n(m-m')dm' - n(m)\int_0^\infty \beta(m,m')n(m')dm' \qquad (8)$$

where the coalescence frequency factor $\beta(m,m') = K$, a constant. The first term in Eq. (8) describes the rate of generation of granules of mass m from masses m' and $m - m'$. The second term is the rate of disappearance of granules of mass m by collision with any other granule. Multiplying by m and integrating with respect to m, the first term in Eq. (8) contributes to the integral in Eq. (6)

$$\frac{K}{2}\int_0^\infty mdm \int_0^m n(m')n(m-m')dm'$$

$$= \frac{K}{2}\int_0^\infty n(m')dm' \int_{m'}^\infty mn(m-m')dm$$

$$= \frac{K}{2}\int_0^\infty n(m')dm' \int_0^\infty (m''+m')n(m'')dm''$$

$$= KMN \tag{9}$$

where we have substituted $m'' = m - m'$,

$$M = \int_0^\infty mn(m)dm \tag{10}$$

which is the total mass of granules per unit volume, and

$$N = \int_0^\infty n(m)dm \tag{11}$$

the total number of granules per unit volume. The second term in Eq. (8) contributes

$$-K\int_0^\infty mn(m)dm \int_0^\infty n(m')dm' = -KMN \tag{12}$$

which cancels exactly with the quantity in Eq. (9). That the condition Eq. (6), although necessary, is not sufficient to establish steady-state conditions is illustrated by choosing K a function of time t. We shall still have cancellation of Eqs. (9) and (10), even though K is time-dependent.

A second example of the steady-state condition is given by the form for $S(m)$ which describes the crushing/layering process, namely (*12*)

$$S(m) = -B(m)n(m) - \frac{d}{dm}(n(m)G(m)) \tag{13}$$

where $B(m)$ is the fraction of granules broken in unit time, and $G(m)$ is the growth function, defined as the rate at which granules growing past mass m pick up crushed material. It is assumed that the only source of layering material is from crushed agglomerates. Multiplying by m and integrating with respect to m, the second term in Eq. (13) contributes

$$-\int_0^\infty mdm \frac{d}{dm}(G(m)n(m)) = \int_0^\infty G(m)n(m)dm \tag{14}$$

integrating by parts and assuming that $mnG \to 0$ as $m \to \infty$. The condition Eq. (6) is

$$\int_0^\infty [G(m) - mB(m)]n(m)dm = 0 \tag{15}$$

which states that the total increase in granule mass due to layering equals the total loss due to crushing.

SIMPLIFIED CONTINUITY EQUATION IN CONICAL DRUM

Under steady-state conditions, Eq. (1) reads

$$\nabla \cdot (n(m)\mathbf{v}(m)) = S(m) \tag{16}$$

This states that the rate of increase due to agglomeration in the number of granules in unit volume and in the mass range $m, m + dm$ is equal to the rate at which these granules are leaving the unit volume, so that the net rate of increase is zero. Consider the volume δV of the agglomerate charge contained in the thin slab between the parallel planes at z, $z + dz$, normal to the axis of the rotating conical drum. Applying the divergence theorem, the integral over the volume δV of the left-hand side of Eq. (16) is transformed to a surface integral, namely

$$\int_{\delta V} \nabla \cdot (n(m)\mathbf{v}(m))dV = \int_{\delta S} n(m)v_n(m)dS \tag{17}$$

where δS denotes the surface bounding the volume δV and $v_n(m)$ is the outward component of the granule velocity normal to the surface δS. On the rim of the slab, where the agglomerate charge has an interface either with the solid drum or the air, $v_n(m) = 0$ (Fig. 2). Hence we need only consider the contribution to the right-hand side of Eq. (17) from the two faces of the slab, which have areas $A(z)$ and $A(z + dz)$. Introduce cylindrical coordinates R, θ, and z, where the z axis coincides with the axis of the conical drum. The granule density n and the component of the granule velocity in the z direction, v_z, depend on the three coordinates R, θ, z, as well as on mass m. The right-hand side of Eq. (17) consists of the integrals over the two faces of the slab, which we write as

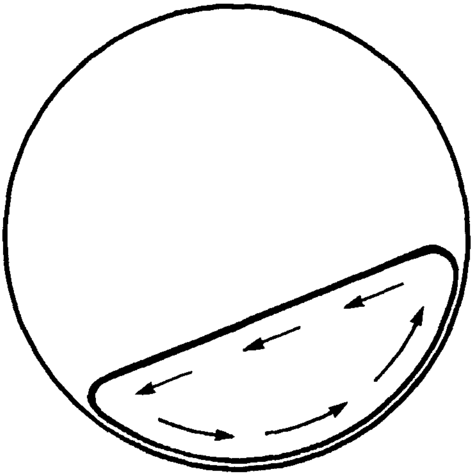

FIG. 2. Cross section of rotating conical drum, showing granular motion.

$$\int_{A(z+dz)} n(m,z+dz,R,\theta)v_z(m,z+dz,R,\theta)dS - \int_{A(z)} n(m,z,R,\theta)v_z(m,z,R,\theta)dS = A(z+dz)(\overline{nv_z})_{z+dz} - A(z)(\overline{nv_z})_z \quad (18)$$

where the overbar denotes the average value of the integrand over the respective area. The right-hand side of Eq. (18) may be written as

$$[A(z+dz) - A(z)](\overline{nv_z})_{z+dz} + A(z)[(\overline{nv_z})_{z+dz} - (\overline{nv_z})_z] = dz\,\frac{dA(z)}{dz}(\overline{nv_z})_z + dzA(z)\frac{d}{dz}(\overline{nv_z})_z \quad (19)$$

neglecting order $(dz)^2$. The integral over over the volume δV of the source term $S(m)$, which also depends on R, θ, and z, is

$$\int_{\delta V} S(m,z,R,\theta)dV = dzA(z)\bar{S}(m,z) \quad (20)$$

where $\bar{S}(m,z)$ is the mean value of $S(m)$ over the area $A(z)$. Equating Eqs. (19) and (20), assuming that the granule density n depends only on m and z and writing $(\bar{v}_z)_z = \bar{v}_z(m,z)$, we get

$$\frac{d}{dz}(n(m,z)\bar{v}_z(m,z)) + \frac{1}{A(z)}\frac{dA(z)}{dz}n(m,z)\bar{v}_z(m,z) = \bar{S}(m,z) \quad (21)$$

We expect $A(z)$ to be nearly proportional to z^2 when Eq. (21) becomes

$$\frac{d}{dz}(n(mz)\bar{v}_z(m,z)) + \frac{2}{z}n(m,z)\bar{v}_z(m,z) = \bar{S}(m,z) \quad (22)$$

By averaging Eq. (6) over any cross-section:

$$\int m\bar{S}(m,z)dm = 0 \quad (23)$$

For a cylindrical drum we may assume $dA(z)/dz = 0$ in Eq. (21).

PROPERTIES OF $n(m,z)$ AND $\bar{S}(m,z)$

Figure 3 shows schematic plots of $n(m,z)$ as a function of the distance z along the axis in the conical drum, under steady-state conditions for

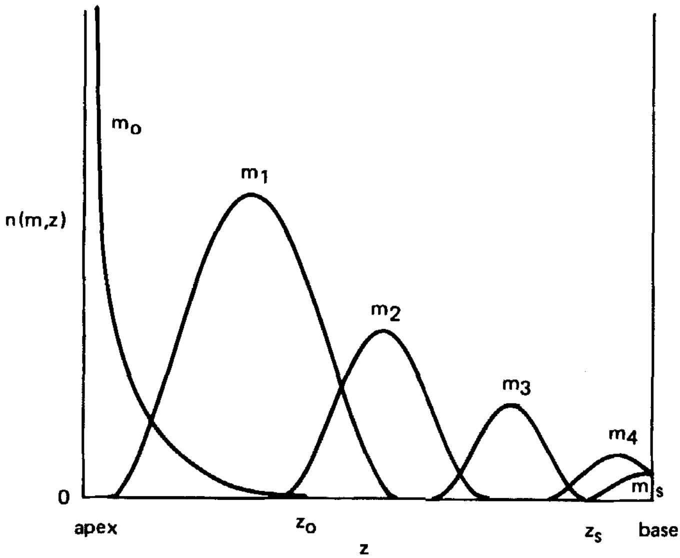

FIG. 3. Schematic representation $n(m,z)$ in conical drum. Constant m contours in (n,z) plane. m_0 = mass of granule feed at apex. m_s = largest mass of granule expelled at base. $m_0 < m_1 < m_2 < m_3 < m_4 < m_s$.

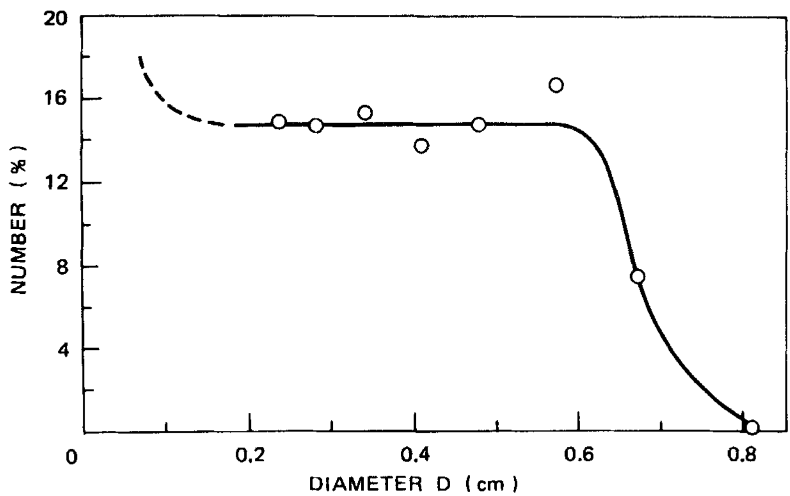

FIG. 4. Percentage of total number of granules in conical drum in each Tyler sieve interval plotted against smallest diameter of interval.

different granule masses. (Strictly, the curves represent $n(m,z)\Delta m$ for a small mass range Δm.) These plots are suggested by the following information on granule distribution which is available from measurements on the spherical agglomeration method of extracting bitumen from oil sands, performed in the rotating conical drum apparatus. A Tyler series of sieves, in which the sieve openings form a geometric progression, having a common ratio $\sqrt[4]{2}$ in the diameter, have been used to separate the granules present in the whole of the conical drum. In a range of sieve diameters from about 0.2 to 0.6 cm, the total number of granules in each sieve interval is very nearly constant. However, this number varies rapidly for diameters smaller than 0.2 cm and at the top end of the sieve scale, as illustrated in Fig. 4. Assuming the granules to be spheres, the striking result from 0.2 to 0.6 cm yields as follows the overall distribution of granule sizes as a function of their diameters. Let $N(D)dD$ be the total number of granules in the range of diameters $D,D + dD$. We require that the integral

$$\int_{D_1}^{2^{1/4}} N^{D_1}(D)dD = \text{constant} \tag{24}$$

independent of D_1. A power law dependence $N(D) \propto D^\gamma$ satisfies this requirement only if the exponent $\gamma = -1$, in which case the progression ratio $2^{1/4}$ can be replaced by any positive number. (Other laws, such as the exponential $N(D) \propto \exp(\gamma D)$ are not suitable.)

The total distribution of granules in the range of D where Eq. (24) holds is

$$N(D)dD = \frac{A}{D} dD \tag{25}$$

where A is a constant. In Fig. 4 the scale of the diameter axis is not uniform but is in geometric progression. On transforming this scale to a uniform one (in arithmetic progression), we replot the horizontal portion of the curve $N(D)$ versus $1/D$ to obtain Fig. 5, which confirms the relation Eq. (25). (Figure 5 was obtained by plotting the cumulant of the experimental number of granules in each interval of the sieve and reading off granule numbers at equal diameter increments.) Introducing the granule mass $m \propto D^3$, it is readily found that the total number of granules in the mass range $m, m + dm$, corresponding to Eq. (25) is

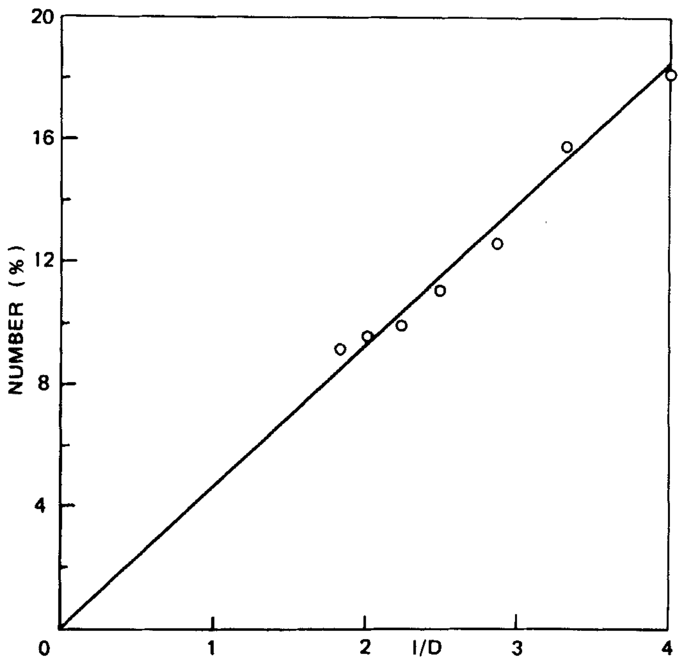

FIG. 5. Percentage of total number of granules in equal intervals of granule diameter plotted as a function of diameter. The points shown were interpolated from the graph of cumulated number percent against sieve diameter.

$$N(m)dm = \frac{A}{3m} dm \tag{26}$$

This inverse power law does not hold for the very small and the very large granule masses. Equation (26) states that if $n(m,z)\Delta m$ is plotted as a function of z in the narrow mass range $m, m + \Delta m$, then the area under this curve decreases with increase in m as $1/m$.

A second piece of information is provided by an analysis, again with Tyler series of sieves, of the size distribution of granules which are discharged from the base of the rotating conical drums. As illustrated in Fig. 6, the distribution resembles a normal one. Visual observation shows that the granules are segregated according to size along the axis of the cone. We expect that distributions from the middle of the conical drum resemble that at the base (Fig. 6) although the granule masses will be smaller. The plots of $n(m,z)$ against z (Fig. 3) are intended to demonstrate size segregation along the cone axis. At the apex, the granules are assumed to have a narrow mass range about m_0. For a given mass, $n(m,z)$ will have a maximum, where

$$\frac{\partial n(m,z)}{\partial z} = 0 \tag{27}$$

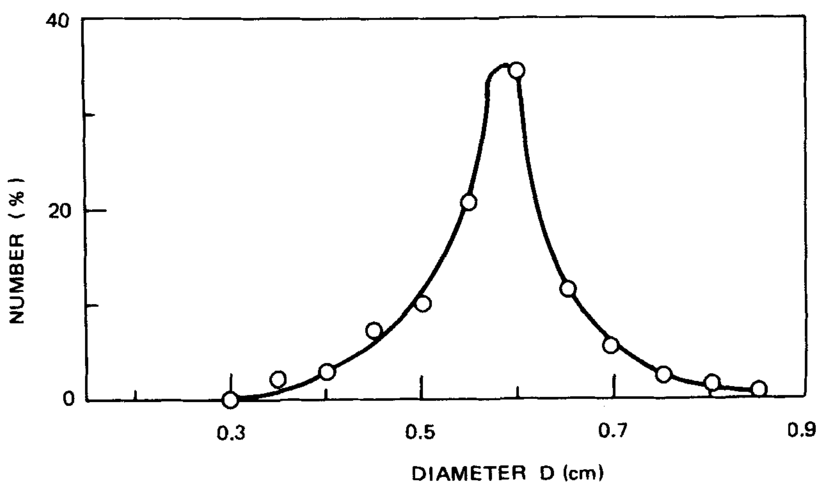

FIG. 6. Percentage of total number of granules discharged at base from conical drum plotted as a function of granule diameter.

which defines a curve in the (m,z) plane. The position of this maximum moves to larger z and smaller $n(m,z)$ (as a result of Eq. 26) with increase in m. For the larger masses, some of the granules are being expelled at the base.

Making use of Fig. 3, we plot constant n contours in the (m,z) plane in Fig. 7. The heavy curve is the locus of the maximum in Fig. 3 defined by Eq. (27). In Fig. 3 the constant m plot for the largest mass $m = m_s$ is so drawn that its maximum is at the base of the cone. In the case where this curve has not reached its maximum at the base, the heavy curve in Fig. 7 would end at the base below $m = m_s$. A curve in the (n,z) plane depicting constant m can be regarded as consisting of two branches, situated on each side of the maximum defined by Eq. (27). Along the z axis ($n = 0$) in Fig. 3 on the low z branch, m increases with z from m_0 at the apex of the cone to m_s. In Fig. 3, let n_s be the value of n at the maximum in the constant m_s curve, which we have placed at the base of the cone. Along a horizontal line $n(m,z) = n_1$ in Fig. 3, where the constant $n_1 < n_s$, again on the low z branch, m increases with m from m_0 to m_s. For constant $n > n_s$, m increases with z from m_0 to terminate on the curve defined by Eq. (27). Turning now to the high z branch, along the z axis ($n = 0$) in Fig. 3, m increases with z, starting at the intersection of the constant m_0 curve with the z axis (say z_0) and terminating at the base. For any $n < n_s$, the constant n curve ends on the curve defined by Eq. (27).

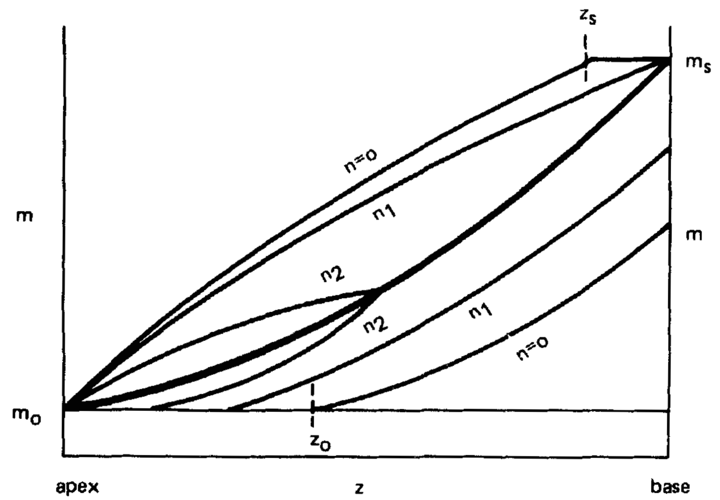

FIG. 7. Schematic form of $n(m,z)$ in conical drum. Constant n contours in (m,z) plane: $n_1 < n_2$.

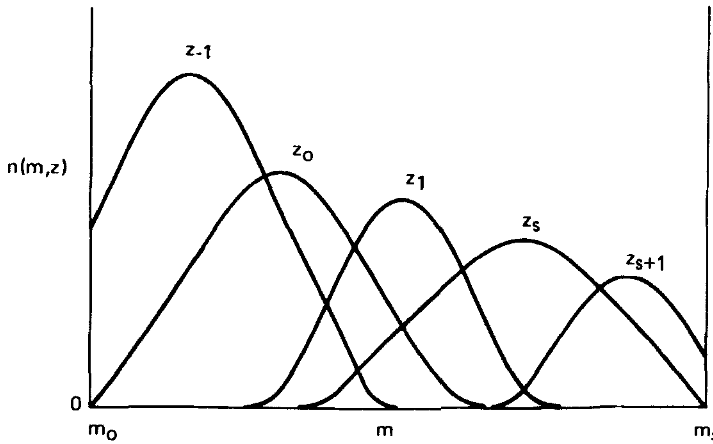

FIG. 8. Schematic form of $n(m,z)$ in conical drum. Constant z contours in (n,m) plane.

By making use of Fig. 7, we plot $n(m,z)$ as a function of m for different z in Fig. 8. To obtain these constant z curves, we examine vertical constant z lines in Fig. 7, considering the variation in n along these lines. The locus of the maxima in the constant z contours in Fig. 7 is given by

$$\frac{\partial n(m,z)}{\partial m} = 0 \tag{28}$$

From Figs. 3, 7, and 8 and the condition Eq. (23), $\bar{S}(m,z)$ must take positive and negative values over the range m_0 to m_s for m at any fixed z. Also, $\bar{S}(m,z)$ must be zero in those ranges of m and z where $n(m,z)$ is zero (see Fig. 7). Consider first a value of z in the middle part of the axis, say at $z = z_1$ in Fig. 7. As m is increased from m_0, $\bar{S}(m,z) = 0$ because $n(m,z) = 0$ until we reach the plot of $n(m,z)$ for fixed $z = z_1$. In the cross-sectional area of the conical drum at $z = z_1$, we can expect the number of smallest granules to be diminishing while the number of largest granules is increasing. This means that with increase in m, $\bar{S}(m,z)$ is at first negative and then positive, returning to zero again when $n(m,z)$ becomes zero. This profile of $\bar{S}(m,z)$ shifts with a change in z, as illustrated in Fig. 9. $\bar{S}(m,z) = 0$ defines a function $m = m(z)$, where m increases with z. Note that because of the weighting factor m in the relation Eq. (23), the area under the negative part of $\bar{S}(m,z)$ exceeds that under the positive part. Also, size segregation is such that the granules become larger and hence their number is reduced with an increase in z. This implies that the magnitude

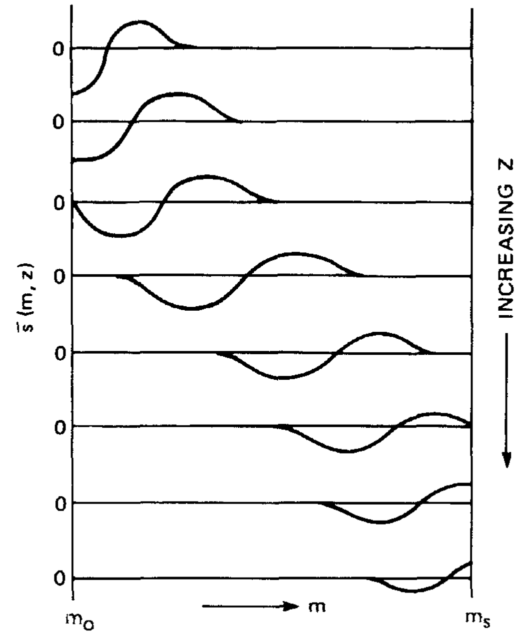

FIG. 9. $\bar{S}(m,z)$ plotted as a function of m for different fixed z.

of $\bar{S}(m,z)$ in Fig. 9 decreases as z increases. Now suppose we keep m fixed and observe the variation of $\bar{S}(m,z)$ with z. Figure 10 is readily obtained from Fig. 9 merely by drawing a set of vertical lines in the direction of increasing z through the plots of $\bar{S}(m,z)$. For the middle range of m, $\bar{S}(m,z)$ will be zero for small and large values of z. In the range of z where a plot of $n(m,z)$ at such intermediate m is drawn in Fig. 4, $\bar{S}(m,z)$ will at first be positive and then negative, with an increase in z. The profile of $\bar{S}(m,z)$ shifts with z as shown in Fig. 10. This behavior of $\bar{S}(m,z)$ as a function of z for different m can also be predicted from Fig. 4.

Further information about $\bar{S}(m,z)$ as a function of z can be obtained by integrating Eq. (22) to obtain

$$\bar{v}_z(m,z) = \frac{1}{z^2 n(m,z)} \int_0^z z^2 \bar{S}(m,z) dz = \frac{I(m,z)}{z^2 n(m,z)} \tag{29}$$

where $\bar{v}_z(m,z)$ must be finite at the apex ($z = 0$) of the conical drum. No meaning is attached to $\bar{v}_z(mz)$ in those regions where $n(m,z) = 0$ and

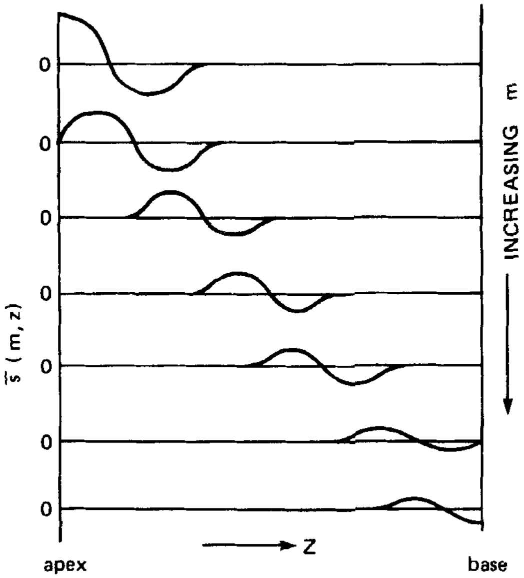

FIG. 10. $\bar{S}(m,z)$ plotted as a function of z for different fixed m.

consequently $\bar{S}(m,z) = 0$. At the cross-over of $\bar{S}(m,z)$ from positive to negative values, where $\bar{S}(m,z) = 0$, $\partial I(m,z)/\delta z = 0$ and $I(m,z)$ is a maximum. Except for large m, where $S(m,z)$ does not return to zero in its negative range with an increase in z, the integral $I(m,z)$ over the whole range of $\bar{S}(m,z)$ must vanish, since $\bar{S}(m,z) = 0$ implies $n(m,z) = 0$. Otherwise, $\bar{v}_z(m,z)$ would become infinite. This tells us that except at the large m in the plot of $\bar{S}(m,z)$ as a function of z, the positive region has a greater area than the negative region. We also conclude that $\bar{v}_z(m,z)$ is always positive. Further information of $\bar{v}_z(m,z)$ requires a study of the momentum balance of the granule motion.

LAYERING—A SPECIAL CASE OF COALESCENCE

Although it has been customary to distinguish layering from coalescence, it is natural to raise the question of the difference, if any, between layering and coalescence of two granules having widely separate sizes. Dropping the factor ½ in the first term in Eq. (8), consider the difference of the two coalescence integrals:

$$\int_0^m \beta(m',m-m')n(m')n(m-m')dm' - n(m)\int_0^\infty \beta(m',m)n(m')dm' \tag{30}$$

We identify m' as the mass of a small granule which is layering on a large granule of mass m. Let us therefore retain only small values of m', say from 0 to Δm in the two integrals and consider

$$\int_0^{\Delta m} n(m')dm'[\beta(m',m-m')n(m-m') - \beta(m',m)n(m)]$$
$$= -\frac{d}{dm}[n(m)\int_0^{\Delta m} m'\beta(m',m)n(m')dm'] \tag{31}$$

very nearly. Comparison of the right-hand side of Eq. (31) with the second term in Eq. (13) suggests that

$$G(m) = \int_0^{\Delta m} m'\beta(m',m)n(m')dm' \tag{32}$$

This result is interpreted as follows: $\beta(m',m)n(m')dm'n(m)dm$ is defined as the number of collisions resulting in coalescence between $n(m')dm'$ and $n(m)dm$ granules in unit time. Replacing $n(m)dm$ by 1, $\beta(m',m)$ $n(m')dm'$ is the corresponding number of collisions between one granule of mass m and $n(m')dm'$ granules in the mass range $m',m' + dm'$ in unit time. Hence $m'\beta(m',m)n(m')dm'$ is the mass of the $n(m')dm'$ granules in the mass range $m',m' + dm'$ layering on the granule of mass m. Assuming that small granules in mass range 0 to Δm are layering on the granule of mass m, Eq. (31) can be written in the familiar form (*12*)

$$G(m) = dm/dt \tag{33}$$

It should be noted that Δm, which remains unspecified, depends on the source of the small granules, for example due to crushing. No restriction is placed on the form of $\beta(m',m)$ in the derivation of Eq. (31).

The omission of the factor ½ in the first integral in Eq. (30) remains to be justified. The number of pairs between $n(m')dm'$ and $n(m-m')$ $d(m-m')$ granules is *half* the product of these granule numbers. Consequently, in Eq. (8) the factor ½ is inserted to ensure that for a *given* m', collisions between granules of mass m' and $m - m'$ are not counted twice. No factor of ½ appears, however, in the second term in Eq. (8) because this term is due to collisions between $n(m)dm$ granules, where m is given, and any other granule. Each member of the given $n(m)dm$ granules can collide with $n(m')dm'$ granules in the mass range

$m',m' + dm'$. Similarly, in layering, we are concerned with the number of collisions between a given granule and $n(m')dm'$ small granules, and a ½ factor is no longer present. The absence of this factor is the difference between layering and coalescence. We shall not discuss further the implications of the result of Eq. (32). This formula for $G(m)$ was obtained earlier by Brock (*13*) in his treatment of the condensation growth process for aerosol particles.

DISCUSSION

In order to make full use of the continuity equation, we require the velocity $\mathbf{v}(m)$ of the granules in the conical drum as well as details of the source term $S(m)$. Information on the velocity $\mathbf{v}(m)$ is derived from the momentum-balance equation for granules of mass range, $m,m + dm$, which corresponds to Eq. (1). In the usual treatment of two-phase flow (*4–9*), the momentum-balance equation for the granules would correspond to Eq. (5), obtained from Eq. (1) by integrating over the distribution of granule sizes. In addition, continuity and momentum-balance equations need to be set up for the continuous phase. Determining the appropriate momentum-balance equations for two-phase systems is much more difficult than establishing continuity equations and, indeed, there is still no general agreement on the correct forms. As a result of the averaging process over volumes large compared with granule size but small on the macroscopic scale, the disperse and continuous phases are each assumed to fill the whole volume of the physical system. This requires introducing stresses in each phase which are defined by constitutive equations containing effective pressures and viscosities. In addition, fluctuation terms are encountered as in turbulence. The presence of agglomeration creates further difficulties. In Part II, we shall examine momentum-balance equations for agglomerating granules under steady-state conditions.

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